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Exploratory Factor Analysis: Conceptual and Methodological Basis

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Review Article

Abstract

The present work reviews the foundations and principal procedures of exploratory factor analysis conceived as an essential method for the construction, adjustment and validation of psychological tests. The article considers the principal strategic decisions that researcher must make during the implementation of this methodology. Additionally, this work presents discussions regarding different conceptual and methodological considerations that exploratory factor analysis presents in each phase of methodology's application.

Resumen

En el presente trabajo se realiza una revisión de los fundamentos y principales procedimientos del análisis factorial exploratorio, un método esencial para la construcción, adaptación y validación de tests psicológicos. El artículo se presenta considerando las principales decisiones estratégicas que debe tomar el investigador durante la implementación del método. Se discuten además las distintas opciones conceptuales y metodológicas que se plantean en cada fase de la aplicación del Análisis Factorial Exploratorio.

Tabla de Contenido

Key Words:
Exploratory Factor Analysis; Psychological Tests; Extraction Methods.

Palabras claves:
Análisis Factorial Exploratorio; Test Psicológicos; Métodos de Extracción.

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1. Introduction

In general terms, Exploratory Factor Analysis (EFA) is the umbrella term used to refer to a set of multivariate interdependence statistical methods whose main purpose is to identify an underlying factor structure within a wide data set. The term EFA may refer either to a set of statistical techniques or to a unique interdependence method (Khan, 2006), which is used to reduce a great number of operational indicators to a lower number of conceptual variables (Blalock, 1966). Although this technique is widely used in the field of social sciences, it is particularly relevant in the psychometric field. Indeed, the EFA is the conclusive step to verify the internal structure of any scale and to select and give a theoretical significance to an initial set of items of a test (Martínez Arias, 1995). This multivariate method allows for clustering variables (e.g., items) that are highly correlated with each other, and whose correlations with another groups' variables (factors) are lower. Even though the variables used are generally

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continuous, it is also possible to apply this method to dichotomous variables (Khan, 2006).

According to Kline (2000), through the EFA, the variability of scores of a set of variables can be explained by a lower number of dimensions or factors. Thus, for example, a great number of items of tests can be reduced to a small number of factors or dimensions (such as verbal ability and extraversion) that confers a theoretical significance to the measure. Each of these factors clusters intercorrelated items which, at the same time, are relatively independent of the remaining set (factors) of items.

Like other statistical techniques, the EFA begins with Galton's works (1889). He suggested the concept of latent trait to explain why a set of variables were related. According to this author, the fact that two variables are related with each other implies that both variables have something in common and something that differentiates them. In this way, the total variance of a variable results from the factors shared with other variables (communality) and the specific factors of the variable (specificity). The EFA logic is based on this idea. It is worth saying that, if a set of variables are correlated with each other, these reciprocal relations are caused by a factor or latent trait in common. Besides, the variance of the variables or measured indicators is partly explained by such factor (Blalock, 1966). With this in mind, Galton (1889) said that it was necessary to develop a technique that would allow discover these factors or underlying latent variables.

The development of the basic principles of EFA could also be attributed to Pearson (1901), who developed the correlation coefficient and outlined the principles on which the principal axis factor analysis is based. However, there is general consensus about considering Spearman (1904) as the creator of the EFA. This British psychologist applied the EFA in order to study the correlations among different ability tests, which were supposed to reflect the underlying intelligence factor. Spearman's research was formalized in the bifactor theory of intelligence. According to this theory, there was a general factor of intelligence (communality of tests) that was partially connected with other specific mental abilities (Figure 1).

Since the 1930s, the EFA is resumed by Thurstone (1947), who consolidated the methodological basis and proposed a reformulation of it.

According to Thurstone's theory, the activities carried out by people depend on certain number of attributes or factors that intervene in different combinations and can be objectively determined by applying the EFA. Thurstone's empirical works led to the discovery of a set of factors involved in the abilities and personality domains.

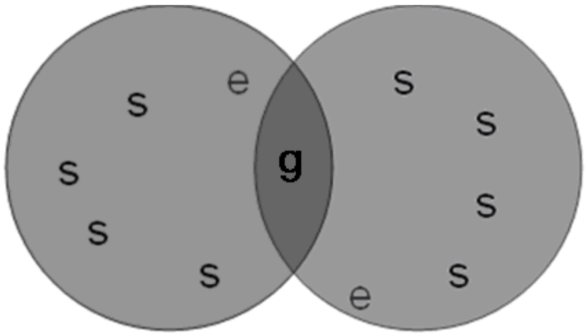

Figure 1. Spearman's Bifactor Theory of Intelligence (adapted from Cohen & Swerdlik, 2006).

Over the years, many authors from different countries resumed the task of developing the technique of the EFA. It is worth highlighting the works of Stephenson, Vermon and Eysenck from England; Kelley, Hottelling, Cattell and Horn from the United States; Meili from Germany, and Rimoldi from Argentina (Yela, 1996). However, beyond the developments and reformulations, the logic of EFA is still the same. Indeed, it is considered that there exists a series of underlying unobserved variables that can be measured by using multiple observable indicators. Thus, perhaps the primary value of this method is that it can replace a great number of indicators that have limited theoretical significance with a smaller number of conceptually meaningful variables (Blalock, 1966).

2. *Assumptions of Factor Analysis*

Before doing an EFA, the fulfillment of a series of exacting statistical assumptions should be verified. If not, the results could be misleading. Specifically, Martínez Arias (1999) points out that before doing an EFA, three main assumptions should be analyzed: normality, linearity and multicollinearity in scores. In addition, the results of the EFA and the statistical analysis designed to evaluate the assumptions that were previously mentioned can be distorted by cases of marginal scores (outliers uni and multivariated). Therefore, it is recommended that an initial exploratory analysis be carried out first in order to detect outlier cases or cases with extreme values.

Outlier cases are those in which extreme values occur on one variable or on a set of variables. This is what makes them differ from the behavior of the rest of the sample (Uriel & Aldas, 2005). Even though not all outlier cases are necessarily problematic, they may turn into observations that distort the results. In order to examine the impact of outlier or marginal cases on the EFA, this method must be considered as part of a correlation matrix between variables, and such correlations must be estimated based on the mean or the average value of such variables. According to Pagano (1998), the mean is a statistical parameter sensitive to extreme values, i.e. an outlying value of the central tendency causes large displacements on the mean. Therefore, if the mean is distorted, the correlation between variables, and thus the EFA will also be affected.

There are different methods to detect outlier or extreme univariate cases. The most used one consists of calculating the typical scores of each variable and considering those cases whose z-scores are outside of the range ±3 as potential outlier cases (Tabachnick & Fidell, 2001). An alternative approach could be to look at the box plots, which display outliers, as isolated points at the end of these plots. Considering that even working with univariate data could generate atypical multivariate combinations, it is recommended to use the Mahalanobis distance (D^2) procedure in order to find multivariate outlier cases. By using this method, it is possible to detect the multivariate outlier cases, which are those cases that exceed the significance threshold ($p < .001$) (Uriel & Aldas, 2005).

Generally, the assumption of normality of the variables has to be verified once it has been detected the existence of outlier cases. While the statistical procedure commonly applied to evaluate the normality of a distribution is the use of goodness-of-fit contrast tests, like statistical Shapiro-Wilk and Kolmorogov-Smirnof, such statistical are too sensitive to small normality deviations when they are used with large samples, as Pérez (2004) proposes. For this reason, it is not advisable to use the contrast statistical as the unique method of normality evaluation. An alternative method consists in estimating the asymmetry and kurtosis indexes, considering that the values inside the ±1.5 threshold indicate minor variations to the normal, and consequently, they turn to be suitable to carry out the EFA (George & Mallery, 2001). Another alternative approximation is the visual analysis of q-q plot graphics; these graphics provide a linearization of the normal distribution and allow to determine whether the compiled data adjust reasonably to a normal distribution.

The assumption of relations linearity becomes fundamental in the EFA, since the values of correlation coefficient can only be interpreted when the relations pattern among the variables is linear (Batista Foguet & Gallart, 2000). In fact, in the case there exist deviations in the linear pattern the magnitude of the correlation coefficients reduces significantly. This assumption can be evaluated by testing visually the matrix diagrams of dispersion. If it is observed that the points are organized along a straight line, the assumption of relations linearity can be maintained (Hair, Anderson, Tatham, & Black, 1999). To evaluate statistically this assumption, a Curve Estimation can be carried out through a Multiple Regression Analysis. This technique was introduced into psychology by Cohen (1978), and it consists in evaluating the nature of the relation between variables by adding powers (linear, quadratic or cubic) to the regression equation and by checking whether those powers improve significantly the prediction or not. If significant results are not achieved by using the linear functions, but they are by applying quadratic functions, it can be determined that there is no linear relation among the variables.

Finally, it is advisable to carry out a multicollinearity diagnostic between variables or items in order to identify elevated or redundant correlations. Even though the EFA technique requires intercorrelation between variables, it is probable that the analysis will weaken and that an unstable factorial solution will be obtained if these variables are superior or equal to .90 (Martinez Arias, 1999). To evaluate the multicollinearity, it is only necesary that the correlation matrix be observed taking into account the existence of values equal or superior to .90. A more precise collinearity analysis can be carried out by tolerance indices and their reciprocal value, the Variance Inflation Factor (VIF). Such measures signal the degree to which each variable is explained by other variables. Small tolerance (lower than .10) and elevated values of VIF (superior to .10) denote high collinearity.

In addition to the compliance with the statistical assumptions required by the EFA, there are some additional requests of prime importance to this analysis. Since the EFA is based on the matrix of intercorrelations and it is not used as a statistical hypothesis test, it is essential that the sample is large to ensure a minor sampling mistake. In fact, if working with small samples, it is more likely that correlations could vary from one sample to another, factors could be unstable and results could be misleading (Blalock, 1966). The factor analysis must be carried out by using large samples, of about 300 participants, in order to obtain useful and rather stable results (Tabachnick & Fidel, 2001). Ideally, there should be at least 10 participants per variable (or item, in the case of tests), and at least five per item (Nunnally & Bernstein, 1995). Additionally, it is advisable to carry out a different factor analysis depending on the sex, when it is dealt with very large samples (Kline, 2000).

After applying the test to the investigation sample, and before starting the factor analysis, it must be determined whether the items are interrelated enough for this method to be successfully applied (Comrey, 1973). There are some statistical tests that can be applied with this end, and the most widespread are the Bartlett's Test of Sphericity and the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy. The Bartlett's Test of Sphericity allows evaluatingthe null hypothesis which confirms that the variables are not correlated. To do this, it compares the intercorrelation matrix of the collected data with an identity matrix in which all the terms in the diagonal are units and the rest of the terms are zeros. If the values obtained from such comparison turn to be significant in a $p < .05$ level, the null hypothesis is rejected and the variables are considered intercorrelated enough to carry out the EFA (Everitt & Wykes, 2001). Since this test can show significant results even if there are no considerable correlations between variables, it is advisable to additionally apply the KMO measure. This is an average index of the terms in the diagonal of the correlation matrix of the anti-image, which contains the negative values of the partial correlation coefficients of the variables. The logic of the KMO index is that if the variables share common factors, the partial correlation coefficients must be small, and consequently, the values of the matrix diagonal must be elevated, which means that if the proportion of large coefficients is elevated in the matrix, there is more interrelation between variables (Sierra Bravo, 1981). The KMO is interpreted similarly as the reliability coefficients, but only with a range from 0 to 1, and considering as appropriate a value equal or superior to .70, which suggests a successful interrelation between items (Hair et al., 1999). Only after having verified the assumptions and the requests of the analysis it is fair to apply the EFA in its different versions.

3. Methods of Factor Number Extraction and Determination

After verifying the compliance of the assumptions, a factor extraction method must be selected. While there are various methods available in the widespread information programs of statistical analysis (e.g., SSPS), in practice the most commonly used methods of exploratory factor analysis are two: Principal Components and Principal Axis (Factoring) Method (Khan, 2006). It is necessary to explain some principles that will allow us to understand the basic differences between both methods. As noted before, the factor analysis is an analytic method of condensation of the total variance in response to variables (or items, in the case of psychological tests). This variance has three main elements: a) the common variance (or communality), which is the proportion of variance of variables that is explained by common factors; b) the specific variance, that is the percentage of particular variance of each variable; and c) the error variance, which is the percentage of non-explained variance, attributable to the measuring error. The Principal Components Method explains the major possible quantity of variance in the collected data. Therefore, this method analyzes the total variance associated with variables, including the specific variance and the error variance.

However, the Principal Axis Factoring Method considers only the common variance between variables, or the covariance, excluding the specific variance and the variance attributable to the measuring error (Tabachnick & Fidell, 2001). The Principal Components Method is easier to understand than the Principal Axis Factoring Method, which is maybe the reason for its popularity, particularly when a significant group of items is analyzed in order to develop new scales or inventories (Merenda, 1997). If the factor analysis is applied to obtain a theoretical solution, non-contaminated by the error and specific variances, the Principal Axis Factoring Method is the most appropriate alternative (Tabachnick & Fidell, 2001). Nevertheless, when the tests have appropriate reliability, the differences among the factor solutions obtained in each method tend to be of less importance (Kline, 2000). An example of this assertiveness can be clearly observed on Table 1, in which the results obtained after carrying out an EFA of a Self-Efficacy Scale for the Performance of University Entrants are represented (Medrano, 2009). The values of the components matrix were obtained by using the Principal Components Method as an extraction method, while the values of the factor matrix were obtained by applying the Principal Axis Factoring Method. Finally, in the last column, the Cronbach's alpha values (α) of the scale are presented if the item is deleted. For this reason, such values must be considered inversely, that is to say, the more α, the less internal consistency is given by the item.

Table 1.
Comparison of the Principal Components Method and the Principal Axis Factoring Method considering the reliability of the items (Cronbach's α values).

Scale Items EAR-I	Component Matrix	Factor Matrix	Alpha value if the item is deleted
Passing the College Entrance Exam	.81	.75	.94
Passing with an average superior to 5	.88	.84	.93
Passing with an average superior to 6	.94	.94	.92
Passing with an average superior to 7	.93	.93	.92
Passing with an average superior to 8	.91	.89	.92
Passing with an average superior to 9	.81	.76	.94

As can be seen, those items that contribute minor internal consistency to the scale, which indicates that are less reliable, show greater differences in the values obtained in each extraction method. On the contrary, the most reliable items present almost no variations among the extraction methods. These results validate Kline's assertion (2000) that differences among factor solutions obtained by each method are minimal when the scales present a high reliability. Nevertheless, some authors suggest that the Principal Components Method is not highly recommendable despite obtaining similar results, since it is a method of data reduction and not an EFA technique. Furthermore, it does not discriminate between common and specific variance, thus tends to inflate the values of the matrix of components (Costello & Osborne, 2005).

Another extraction method of factors that is worth considering is the Method of Maximum Probability (MP), or also known as Maximum Likelihood Method. Although the method of MP is less used in the explanatory factor analysis (Kahn, 2006), plenty of authors agree that MP is the best choice when data present a multivariate normal distribution (Byrne, 2001; Costello & Osborne, 2005). The main benefit of MP is that allows to estimate the statistical significance of the factorial weights and produces confidence intervals from them. Nevertheless, when data distribution distance from a multivariate normal distribution, it is preferable not to use MP; maybe for that reason, in practice it is one of the extraction methods less used (Kahn, 2006). In short, the Principal Axis Factoring Methods and MP are the methods that provide better results and the choice of one or the other will depend on the distribution of the data collected; the Principal Axis Factoring Method is the recommended method when the assumption of multivariate normality is violated, and MP is the recommended method when such assumption is respected (Costello & Osborne, 2005).

The extraction of the right number of factors is one of the most problematic decisions of the factor analysis (Cattell, 1966). The usage of a unique criterion may lead to overestimate or underestimate the real number of factors, and for this reason, it is recommended to use a set of criteria to identify the number of the underlying factors on the psychological scales. If the option is to extract more or less factors (over and under-extraction), the over-extraction is less risky because it is less likely to produce mistakes on the measurement (Reise, Waller, & Comrey, 2000). Nevertheless, the decision about the number of factors that will be extracted shall always be supported by empirical evidence. A widely used method, which appears in the SPSS program, is the Kaiser rule of extraction of factors with eigenvalues superior to 1 (Kaiser, 1960). The square of the correlation between a variable and a factor is the proportion of variance explained by such variable. If all the squares of the variable factorial weights in one factor (column of the factorial matrix) are added up, the eigenvalue of that factor is obtained, which expresses the magnitude of variance explained by that factor. The point of cut of 1 is determined because variables are standardized with the variance equal to 1 and it would be inappropriate to interpret a factor that explains less variance than the variance explained by a particular variable (Kahn, 2006). If we divide the eigenvalue of a factor by the number of variables, and we multiply that value by 100, we obtain the variance percentage explained by that particular factor.

The main problem of this rule is that it generally leads to the extraction of too many factors, particularly in tests with plenty of items (50 or more). Another criterion for extraction is the percentage of variance explained by the factorial structure obtained (accumulated variance of factors extracted in set). In this case, it is recommended that the factorial solution explains, at least, a 50% of the total variability of the test response (Merenda, 1997).

The percentage of explanation of variance could be a necessary condition, but in practice it does not constitute a decisive criterion, because we can have many alternative factorial solutions with appropriate percentages of variance explained and, consequently, we will not know which one to choose. In any case, the Kaiser rule and the percentage of variance explained are complementary procedures, but they are not essential in the majority of cases.

The criterion of extraction of factors most widely used nowadays is the scree test or scree graph (Cattell, 1966). The scree test a graphic representation of the magnitude of eigenvalues, and it contributes to identify the ideal number of factors that should be extracted. On the horizontal axis, or ordinate, the eigenvalues are represented, and on the vertical axis, or abscissa, the number of factors. On the resulting graph, a base straight line is drawn at the same height of the last eigenvalues (the smaller ones), and those that are above that base line will indicate the number of factors that will be retained. Cattell (1966) called this graph "scree" due to its resemblance to the profile of a mountainside, where the waste rock of the base is similar to the irrelevant factors of the solution, metaphorically not solid. The scree test is a procedure with a component of subjectivity, but its appropriate reliability has been verified (Kline, 2000). In general, the point of cut for the number of factors that will be extracted is determined by the first change of slope in the graph. The residual eigenvalues are located to the right of the graph; they form a slightly inclined plain. In contrast, the eigenvalues that explain most part of the variance are located to the left and they form a steep slope (Figure 2).

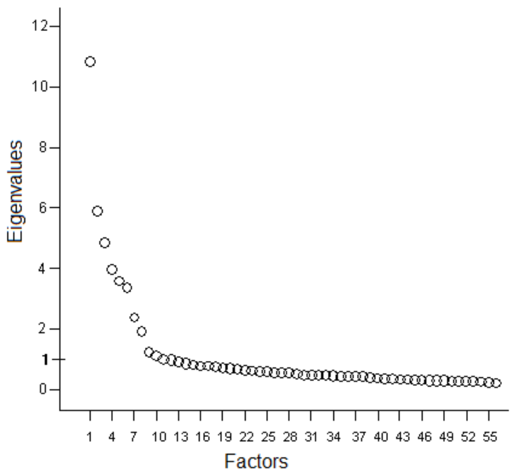

Figure 2. Scree Test

Figure 2 shows the sedimentation graph obtained from the EFA from the "Inventario de Autoeficacia para Inteligencias Múltiples" (Pérez, Beltramino & Cupani, 2003). In general, it is recommended to review the graph from left to right until the point of inflection in which the eigenvalues stop forming a slope and start generating a slightly inclined fall is located. This fall is represented in the dotted line and it is called criterion of contrast fall (Hair et al. 1999). As can be seen, even though the Kaiser rule indicated the existence of 12 factors (dotted line), the scree test suggests that only 8 of them should be interpreted since the fall of the graph is interrupted from the ninth eigenvalue on. In most cases the scree test is reliable, but sometimes it results difficult to determine the exact number of factors by just seeing the graph, especially when there are plenty of factors or changes of direction of the slope.

Horn (1965) proposed another method, the parallel analysis, which seems to be one of the best +alternatives to decide the number of factors that will be extracted. The parallel analysis generates eigenvalues of a matrix of random data but with the same number of variables and cases as the original matrix. Even though the parallel analysis cannot be developed from the usual statistic programs (e.g. SPSS, SAS), Thompson and Daniel (1996) developed a syntax that can be executed simply from SPSS. In the parallel analysis, the eigenvalue of each factor in the real data is compared to the corresponding eigenvalue of the random data on a chart. To decide the number of factors that will be extracted, the eigenvalue of the real data with magnitude superior to the eigenvalue of the simulated data is identified. If, for example, the third eigenvalue of the real data has a magnitude of 3.131 and the third value of the simulated data has a magnitude of 2.431, and the fourth eigenvalue is superior in the data simulated, three factors must be interpreted. The logic of the procedure is that only real factors that explain more variance than the random must be interpreted (Kahn, 2006).

4. Rotation and Interpretation of Factors

The initial result of the factor analysis is a factor matrix not rotated, that is to say, it is the matrix of correlations between variables and factors. This initial factor matrix is difficult to interpret and, in almost all the cases where more than one factor is extracted, it is essential to obtain an additional matrix of rotated factors (Carroll, 1953). Consequently, after extracting the initial factors, these initial factors are subject to a procedure called rotation (when there is more than one factor in the solution). The term rotation comes from the graphic representation and the geometrics of the factor analysis. Indeed, factors can be represented as reference axis and the factorial weights (correlations) of each variable can be indicated in the corresponding axis. The rotation is carried out so as the factor solution approximates to what it is determined as simple structure; it is worth mentioning that each item has a correlation that is close to 1, that could be possible with one of the factors and correlations close to 0 with the remaining factors. The researcher rotates the factors with the purpose of eliminating the important negative correlations and reducing the number of correlations of each item in the diverse factors. Naturally, simple structures are never found in empiric data, but a solution approximated to that theoretical concept is reached. Rotations can be orthogonal or oblique; and two widely used methods are Varimax (Kaiser, 1958) and Promax (Gorsuch, 1983), respectively; although there are others available in the statistic programs (e.g., SPSS). Solutions provided by the oblique rotation methods are more consistent with the structure of the psychological variables that are generally interrelated. Absolute orthogonality is only theoretical and, for practical reasons, it is understood that a solution is orthogonal when all correlations of factors are lower than .32. Tabachinick and Fidell (2001) propose to do an initial oblique rotation as a filter (e.g., Promax), and to obtain the correlation matrix between factors. If we notice any correlation exceeding .32 between factors, we should choose an oblique rotation. Otherwise, we should choose an orthogonal one. Following these recommendations, Medrano (2009) did an EFA of the Positive and Negative Scale of Affectivity (PANAS). When applying a Promax rotation at first instance, it was noticed that the underlying factors had an invert correlation of r = -.33. Based on that, we continued with the Promax oblique rotation. The obtained results are shown in Table 2.

Rotations place the variables closer to the designed factors to explain them; they concentrate the variance of variables on fewer factors and generally provide a means to facilitate the interpretation of the factor solution obtained (Kasier, 1958). Today, there are several executable algorithms in the statistical packages that generate the rotated matrix without resorting to graphic rotation procedures (Thompson, 2004). With these algebraic operations (multiplication of unrotated coefficients by a set of derived constants by means of trigonometric functions), the structure of the factor matrix is modified and easier to understand due to the increase of the extreme positive correlations (low and high), approaching the simple ideal structure that we have mentioned. The analytic rotation procedures have replaced the geometric ones because of their simplicity (executed by software) and objectivity (it is more difficult to achieve identical results among several researchers when graphic rotation takes place). Correlations between item and factor should be at least of .35 and there should not be correlation higher than .30 of that variable with another factor to obtain an estimate solution to the simple structure. If not, we shall be retaining complex items, as well as unsatisfactory and difficult to interpret factor solutions.

It should be taken into account that if we have used an oblique rotation (e.g., Promax), we will not obtain a rotated matrix (the same case as orthogonal rotation) but two, which are called structure and configuration matrix. The correlations of each variable are present in the structure matrix with the factor or structural coefficients. However, in the configuration matrix the observed coefficients are analogous to the beta coefficients of the Multiple Regression Analysis. The configuration coefficients indicate the relative importance of each factor to explain the individual score of each variable, controlling the remaining factors. Most of researchers interpret the configuration matrix because it is easier (Tabachnick & Fidell, 2001), but it is advisable to pay attention to both matrixes so as to better interpret results (Thompson, 2004).

Table 2.
Promax rotation of PANAS scale, comparison of coefficients of the unrotated matrix, configurational matrix and structure matrix.

	Unrotated Factor Matrix		Configurational Matrix		Structure Matrix	
PANAS scale	Factor 1	Factor 2	Factor 1	Factor2	Factor 1	Factor 2
Sorrowful	.44	.22	.48	.12	.47	.10
Guilty	.53	.05	.53	-.06	.53	-.08
Scared	.78	.28	.83	.11	.82	.08
Irritable	.52	-.05	.50	-.16	.50	-.18
Ashamed	.57	.05	.56	-.07	.56	-.09
Nervous	.67	.20	.70	.05	.70	.03
Uneasy	.70	.04	.69	-.12	.69	-.14
Fearful	.79	.23	.82	.05	.82	.02
Strong	-.27	.52	-.13	.57	-.15	.57
Enthusiastic	-.14	.70	.04	.71	.02	.71
Proud	-.16	.64	.01	.66	-.02	.66
Inspired	-.08	.60	.08	.61	.06	.60
Determined	-.40	.46	-.28	.54	-.29	.55
Attentive	-.08	.68	.09	.69	.07	.68
Active	-.27	.67	-.10	.71	-.12	.71

The final task of the factor analysis is to interpret and define factors. This is achieved by examining the rotated matrix (provided that there are more than one factor) the high and low patterns of correlation of each variable with its different factors and using specially the theoretical knowledge about the variables included in the analysis. In the example of Table 2, it can be observed that the factor 1 has higher correlations with items such as: fearful, scared, nervous and guilty. Thus, the factor could be interpreted as “Negative Affect”. On the other hand, the second factor has elevated correlations with items such as: enthusiastic, attentive, active and proud. Thus, the second factor could be interpreted as “Positive Affect”.

In general, it is recommended that each factor have at least four items with equal or higher correlations to .40 to be interpreted. Furthermore, it should be taken into account the higher item–factor correlations to infer the name of each factor (Glutting, 2002). When the realized rotation has been oblique, it is possible to continue with the factor analysis and to obtain “factors from factors." In other words, we can make a factor analysis and draw oblique factors of initial order to make a factor analysis of the correlation matrix between factors and to derive second order or higher factors.

5. Conclusions

In this article, some of the steps and critical decisions in the development of an EFA have been synthetically revised, i.e., assumptions required, factor extraction methods, criteria to determine the number of factors and rotation methods and interpretation of factors. It is important to notice that generally the exploratory factor analysis is the most classic and well known statistical procedure to examine the relation between a set of observable variables and unobservable underlying factors. Thus, this method of interdependence identifies the existence of the underlying factors of important theoretical value starting from relations between observable variables. However, it is important to notice one limitation of EFA. As mentioned before, this technique is supposed to delimit the number of indicators that presumably measure one construct empirically, that is to say, the EFA does not require a specification previous to the theoretical model. In general, users of EFA limit themselves to hypothesize the number of factors that are expected to obtain and if they will or will not be related (Pérez-Gil, Chacón Moscoso, & Moreno Rodríguez, 2000). Consequently, the exclusive use of EFA represents a weak approximation to the definition or validation of a construct. As Byrne (2001) says, EFA should be complemented with a subsequent Confirmatory Factor Analysis (CFA). In fact, the exclusive use of EFA may lead us to obtain only empiric structures, depending on the samples and selected items, not easily replicable.

The CFA is a complementary and appropriate approximation to the EFA when researchers have certain knowledge of the underlying theoretical structure. Thus, based on theoretical and empiric criteria, the relationships between the observable and latent variables are postulated a priori to evaluate later their statistic meaning and the adjustment of the model proposed to collected data (Batista Foguet & Gallart, 2000). Beyond mathematic and statistic differences between EFA and CFA, the main difference is that confirmatory approximation does not realize on a vacuum, but is found within a theory that manages the definition of construct. This approximation goes from theory to practice, which is why it is a stronger approximation of the definition or validity of construct. Nowadays, even when CFA procedures are very well developed, EFA is still used for confirmatory reasons (Pérez-Gil, Chacón Moscoso, & Moreno Rodríguez, 2000). It is important to notice that EFA is a valid technique with exploratory goals that will lead to random and likely unstable results if the construct to validate or define is unknown, since this procedure depends entirely on the circumstances and collected data. In this sense, Eynseck says that "the factor analysis is a good servant but a bad master".

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