Some remarks on Navajo geometry and piagetian genetic theory

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Abstract

Some of the basic findings of Piaget's study on spatial knowledge in Genevan children are contrasted to Navajo spatial knowledge: there is no unique spatial «primitive» in Navajo knowledge and each and every notion is constituted in a different way than its Genevan correlate. The total system of Navajo knowledge shows a structure of interrelated and codetermining notions which differs markedly from the unilinear logical-deductive structure in the Piagetian model. In the second half of the paper the investigation turns to education and schooling. Here a position is taken against the (Piaget-based) rationalistic education policy one finds in the New Math-movement. Instead, some of the principles of Freudenthal are explored in a non-western setting: children should reinvent geometry on their own, in the terms of their culture. To that end, a curriculum for geometry teaching in a Navajo bicultural school is presented, focusing on the schooling in geometric concepts within the Navajo language and with full respect for the Navajo spatial representations in pre-school children.

Keywords: *Piaget, Space, Geometry, Navajo.*

Observaciones sobre la geometría de los indios navajo y la teoría genética piagetiana

Resumen

Algunos de los hallazgos básicos que se encuentran en los estudios de Piaget sobre el conocimiento del espacio en niños ginebrinos son confrontados con el conocimiento espacial de los navajo: no hay un «primitivo» espacial único en el conocimiento navajo y todas y cada una de sus nociones están constituidas de manera diferente a su correlato ginebrino. La segunda parte del trabajo se centra en la educación y la escolarización, rebatiendo los principios de la política educativa racionalista basada en Piaget que podemos encontrar en el movimiento de la nueva matemática y explorando los principios de Freudenthal en un escenario no occidental. Se presenta un curriculum para la enseñanza de la geometría en una escuela navajo bicultural centrado en los conceptos geométricos propios del lenguaje navajo y respetando las representaciones espaciales de la cultura navajo en los niños en edad preescolar.

Palabras clave: *Piaget, Espacio, Geometría, Navajo.*

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1. SPATIAL REPRESENTATION OF NAVAJOS

The Navajo world view

In a previous work (Pinxten et al., 1983) the world view of Navajo Indians was extensively represented. Since the basic orientation of the Navajo view differs from the western one and since the teaching of geometry is bound to draw on spatial and general natural philosophical intuitions in the student's culture (cf. below Part II), it is deemed relevant to sum up the main points of the previous analysis.

In general, five main characteristics of the Navajo world view were seen as highly relevant in order to understand Navajo spatial representations.

a) The structure of the world

The Navajo world is conceived as a combination of two dishlike phenomena: earth is seen as a dishlike form stretched out in all four directions, and heaven is a dishlike form hanging above earth upside down.

The earth, like the hooghan (house) and the wedding basket, is said to be closed on all sides, except for the east.

b) The dynamic nature of the universe

Everything is inhabited by some sort of power or structuring principle (Navajos speak of winds: nich'i), which keeps a thing standing throughout changes and against the forces that work on it (other winds, water, etc...). In fact, the substance out of which reality is constituted in the Navajo conviction is events, changes, processes, much more than objects or states. This profoundly dynamic view on reality is guided, as it were, by the fact that Navajo language consists overwhelmingly out of verbs and verbal constructions. (Young and Morgan, 1980).

While investigating the part/whole distinction in Navajo language Pinxten et al. (1983) found that this distinction was almost absent: Navajos express functional relationships rather than part/whole ones, which is of course more congenial with their view on the world. In the structure of the language (the overwhelming amount of verbal forms), in the behavior (the constant reference to continuity in existence and to constant change in everything) and in the cognitive representations the dynamic view is the most powerful and the «natural» way to deal with things. This should be kept in mind for education as well.

c) Boundedness of the world: center-periphery organization

The Navajo space is bounded in each cardinal direction by a Sacred Mountain. The world is thus bordered in a horizontal plane. Similary, the world is bounded by the shell of the sky and by the shell of the earth, thus realizing a neatly bounded bowl.

The world, just like the hooghan and the wedding basket are constructed around a center. This center goes from the nadir or center of the earth up into the zenith or center of heaven. People point to a particular star formation to locate the center of the sky and to a specific mountainous rock (Huérfano Peak) to identify the center of the earth. Behavioral rules (sleep around the center of the hooghan in clockwise formation) and beliefs (ceremonies are performed in the center of the hooghan) all grant this crucial role to the center notion.

d) Closedness of the world: the notion of order

According to mythohlogy everything in the Navajo universe has been there since this universe was created. This particularity of Navajo world view is what is referred to here as the «closedness» of the Navajo world (somewhat in the sense of Koyré's understanding of open and closed world views, 1957).

All and everything in the Navajo world is fitting in a preconceived ant to man unknown network of orderly relationships. Navajos speak about the orderliness, harmony or beaty of nature (hozhoon) in this respect. The concept is a very basic one and it has been the subject of study for several major researchers on Navajo (e.g. Haile, 1947; Witherspoon, 1977; Farella, 1984). It should be understood that order is thus projected into nature, or is presupposed to exist in nature without however its been known by the human knower.

e) The interrelatedness of everything in the Navajo universe

This notion refers to some extent to the former one. Nothing in the Navajo world is on its own. Everything is embedded in a complex network, by means of which it is linked to many other things and ultimately to the whole. The Navajo universe is homogeneous throughout and any disturbance in one particular place or time will have effects on everything else and for a considerable time. Human beings are as much embedded in the englobing network of relationships as any other aspect of the world. Interrelatedness is first and foremost expressed in spatial terms: things are «placed» relative to each other.

Navajo spatial knowledge

The study of spatial knowledge in the Navajo culture concentrated mainly, though not exclusively, on linguistic material. It is necessary to give a short overview of the Navajo model of space in order to understand the further part of this paper.

a) The semantic model

The study of Navajo space can be situated, from the point of view of methodolody, in the branch of cognitive anthropology. However, the model proposed and the criteria of semantic analysis used are different from the standard ones within cognitive anthropology (cf. Pinxten, 1977, 1983). The criteria for semantic analysis allow for synthetic study only and grant as much room as possible to the native insights on every step of the work. Thus, I abide with the Piagetian focus on synthetic propositions (Apostel et al., 1957), as worked out by Quine (1960).

The final result of semantic analysis (the identification of native categories and their representation in a cognitive model) is thus not understood to be definitive or to consist of full definitions in a real sense. Both consultant and ethnographer are conscious of the gap of untranslatability and they will consequently strive for the ultimate or optimal representation or for the least alienating cognitive category possible. It is therefore useful and careful to refrain from complete descriptions of native categories and to settle down for incomplete representations. Each and every category of Navajo space is composed of two subsets of «features» or components: Const. I and Const. H. The first formula (Is a constituent of) consists of a list of spatial (and some non-spatial) components, identified by Navajos, for which the category paraphrased is acting as a constituent or defining element. The second formula (Has as constituents) consists of the list of all components identified by Navajos which make up the meaning or the category paraphrased in an incomplete, but sufficient way. Each category is an abstract construct which refers to and is practically «filled in» by a set of terms in the language. (For example, a set of some twenty terms has to do with «overlapping»: each term gives a particular and exclusive aspect of overlapping, but the set of them defines the culturally relevant notion of «overlapping» in Navajo, i.e. the category.)

b) Spatial knowledge

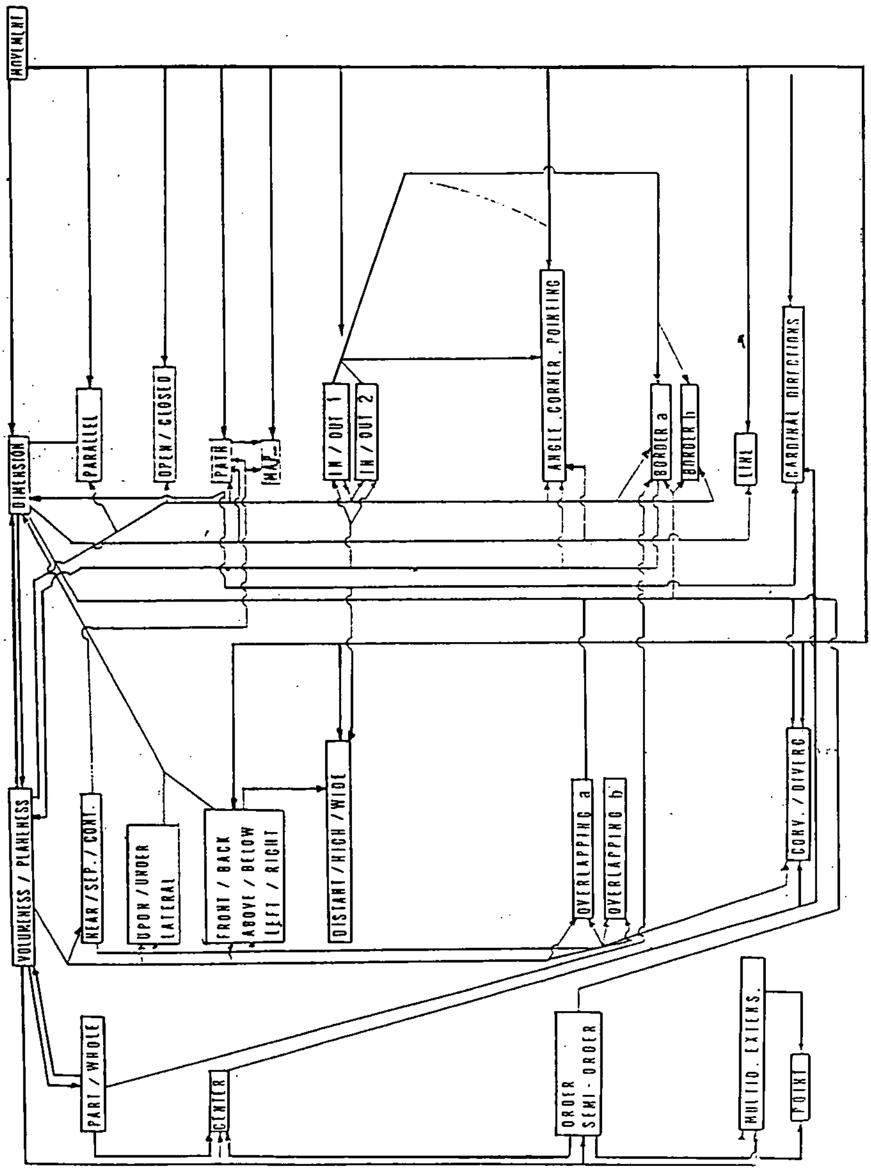
An example will make clear how my theoretical positions are supported by empirical date. The qualifying notion «near» is a good example (Pinxten et al., 1983: 52-55). In the field we gathered linguistic data about all possible way to express «nearness»: we picked up terms, had them use in several sentences each, discussed their overlap or contrast in meaning with other terms, asked for folk definitions, and so on. We observed people exemplifying «nearness» in gestures and in spatial relationships between objects and people. Finally, we sought to contrast «Navajo nearness» (on the basis of their terminology) from e.g. «Navajo distance» (on the basis of another set of terms). This procedure was followed for over 500 terms and scores of observational situations.

With respect to «nearness» the following notions emerged: Navajo distinguish between relative and absolute nearness. In the case of absolute nearness a spatial relationship is described between two objects; with relative nearness the spatial relationship between an (outside) object and ego (the speaker, the perceiver) is indicated. Absolute nearness is expressed e.g. by means of terms such as *áhání* and *binaa.* Relative nearness will be expressed by using e.g. *ch'į́, whóshdę́ę́* and *t'áá áhą́ą́digo* in descriptions. The notion of «near» has the Navajo notions of «volumeness/planeness» (i.e., having extention in three or two dimensions) as semantic constituents. On the other hand, it codetermines the meaning of Navajo «overlapping», «convergence/divergence» and «up/under».

The graphic representation enclosed gives an overview of the total cognitive domain of Navajo space (Figure 1).

I will only treat the three most dominant and thus most basic notions in Navajo spatial representation. They are: «volumeness/planeness», «dimension» and «movement». These notions turned out to be more basic than others, since they are implied in the semantic identification of many other notions. Their semantic content (the Navajo understanding of each of them) is represented in the graphic model as well: the arrows pointing

FIGURA I



Versión española de los textos de la Figura:
Fila superior: Voluminosidad/planitud; Dimensión; Movimiento.
1.ª columna: Parte/totalidad; Centro; Orden, Semi-orden; Extensión multid.; Punto.
2.ª columna: Cerca/Sep/Cont.; Encima/debajo, lateral; Delante/detrás, arriba/abajo, izda/dcha; Distancia/Alto/Ancho; Solapar a; Solapar b; Conv. div.
3.ª columna: Paralelo; Abierto/cerrado; Vía; Mapa; Dentro fuera 1, 2; Angulo, esquina, señalar; Límite a, b; Línea; Direcciones cardinales.

towards these notions indicate the semantic components that constitute the concepts. Each of these is distilled from a large set of terms (ten for each of them), which refer directly of indirectly to the notions themselves: there is not a single label or direct reference to «dimension» in Navajo, but the notion is present in many composites (backward, forward, etc.); there are large sets of verbstems and other verbal expressions denoting «having expansion in all directions» (volumeness), «having expansion in a planelike fashion, flatwise» (planeness) and «movement» (displacement, rotation, translation, etc...).

My general intuition for the construction of a geometry curriculum is that is would be wise to start with these intuitive or broadly topological notions and to retrace the global field of Navajo space from these very broad notions out.

Volumeness/planeness: in an oversimplification we can say that Navajo concepts of volumeness and planeness describe, in a way, the basic Navajo ontology. Thus the basic set of eleven classificatory verbstems (to be refined and multiplied at will) details eleven mutually exclusive and incomparable ways of manipulating or acting on phenomena: e.g. single multidimensional items with a solid appearance are delineated through the use of the *si'a* stem, while a mass of undifferentiated phenomena with considerable extension will be described in terms of *sighi,* and so on. «Objects» in the western conceptualisation can travel from one to another category if they are to be handled in a bundle, as a stack, in a container, and so on. It should be clear that the domain of what is called «volumeness/planeness» here thus covers all non-metric and very rough or intuitive volumes and planes, which are distinguished in the cultural knowledge system of the Navajos.

Movement: Witherspoon (1977) can be quoted to have said that the verb «to go» is for Navajo language and thought what the verb «to be» would represent for westerners. One finds a practically endless serie of forms and aspects of «to go» and it is clear that any phenomenon in the Navajo world will at some point or other be subject to movement. The myriad of terms denoting movement in different senses (from actually moving back and forth, displacing and so on up to rotating and vibrating) can be seen as ever so many aspects of «movement» in the Navajo view.

Dimension: neither the abstract notion of «dimension» nor any of the three spatial dimensions are labeled as such in the language. Rather, the three dimensions are formed as a composite of two parts: in front of + in the back of, above + below, and left + right. In consequence, the notion of «dimension» used in this paper is an abstraction of what was recognized as an implicit concept in Navajo.

Primarily on the basis of these three irreducible spatial notions, and in particular combinations which are exclusive for each of the set of other spatial notions found in Navajo, a model of native spatial representation is built.

For a better comprehension of the educational procedure it is important to be more explicit about the notions of «topology» and «geometry» in this and the following sections. In their path breaking work Piaget & Inhelder (1947) found that Genevan children structured their spatial experiences in three consecutive systems of notions: up to ca. 7 years of age children ordered the space according to «topological» concepts (near, separate, overlapping, bordes, etc.) linked directly to manipulatory exploration of objects and environments. Until the age of 11 or 12 children then develop so-called projective geometric concepts (dimensions, path, etc...). Finally, at the level of formal reasoning, genuine Euclidean notions are learned: e.g. separation is converted into metric distance, shapes are defined in terms of geometric figures (f. ex., square, triangle, etc...). The development from the first topological notions right up to the Euclidean notions in a unilinear and logically deductive process of «unfolding» of concepts, according to the Piaget paradigm.

A basic problem arises: is this unilinear line of development (with the three stages of geometry) universal? Piaget did not address this question. My answer to this question is varied, though overall negative. It is negative in the sense that different concepts —i.e., with different constituents—, obtain in Navajo spatial knowledge (see Figure 1), and that the neat-three-layered structure does not appear at all in the Navajo case. Nonetheless, Navajo correlates of all spatial notions in the Genevan spatial knowledge system can be detected, but their contents (their constituents) and their place in an englobing ordered system of concepts differ markedly.

A further and important contrast with Piaget's model is that ethnographic data on Navajo space point to three mutually irreducible basic actions, whereas Piaget finds a single «primitive» in western spatial thought (neighborhood: Piaget & Inhelder, 1947: 6,3) on the basis of its genetic primacy. Other spatial notions (in the Navajo system) will not be detailed here (cf. Pinxten et al., 1983). For all of them, however, the basic emphasis on synthesis (instead of analytical deduction) in the constitution of notions is safeguarded. I thus share Piaget's important finding: where he stressed this process of concept formation in the genetic development of spatial notions in the western child (Piaget, 1969: 421), I extrapolate the principle to a non-genetic cross-cultural context.

I will take this preschool Navajo spatial knowledge system as a point of departure for geometric education. It should be clear that most so-called projective and metric notions of the Navajo system (the logically most sophiscated ones) will not be presupposed in the child's mind at the primary level (though some, e.g. dimension, clearly are).

2. EDUCATIONAL AND TECHNICAL PROBLEMS

It is impossible in the present study to extensively discuss the different approaches to geometry teaching and its psychological prerequisites. I will only sketch a few lines in the context of two main perspectives: Freudenthal's view (drawing on the intuitionistic school of mathematics) and Piaget's view.

On the basis of the fundamental work on the child's conception of space (Piaget & Inhelder, 1947) Piaget concludes that there is a specific, logical pattern of development of spatial representations in the child (going from topological, over projective, to metric notions). A fundamental point is that the so-called «topological» notions are primary to any other: «aux niveaux représentatifs les plus élémentaires, où manque ainsi toute métrique euclidienne et toute coordination projective..., on constate la présence d'un certain nombre d'intuitions topologiques fondamentales» (Piaget, 1969: 419). Recent research points out that Piaget's notion of «topology» is akward (although the representational phenomena it denotes in the child are confirmed), and that the primacy thesis cannot be withheld (cf. Darke, 1982).

Piaget's model of the growth of geometric notions is founded (as for all aspects of genetic psychology) on an evolutionary epistemological basis: «For Piaget, mathematical inventions must also originate from structures in the population». (Easley, 1982: 144). In Piaget's view mathematics theachers should pay attention to the psychological developmental processes in their children (idem, 169). The Genevan outlook thus led to the acceptance of what I would call a «moderate a priori universalism»: structures have to develop in a certain order, but their development is not automatic, but rather maturational. The emphasis on universalism remains, as is seen in the cross-cultural studies (where only a retardation of two or three years is granted for non-western children, but certainly no «alternative» development: e.g. Berry & Dasen, 1974). Piaget's universalism is aprioristic, since the biological paradigm is guiding him; however, it is of a moderate kind, since «maturation» implies some impact of the environment through development at the ontogenetic level.

In contradistinction with these epistemological choices I came to favor, —on the basis of field work on cognitive processes of spatial representation—, the notion of «a posteriori universalism» (Pinxten et al., 1983). That is, after the fact and from a cross-cultural comparison, it may emerge that certain cognitive features are universal to some extent. The universals are much more conditional and much less automatic than in Piaget's approach. I want to safeguard the minimal universalistic elements I found and develop or sophisticate the alternative of culture-specific spatial representations. I expand and reformulate Piaget's position in order to justify the educational strategy I follow: instead of claiming that universal knowledge of the genetic development of formal notions is a necessary foundation for their understanding in scientific contexts (Piaget, 1969) and thus, implicitly, for their development through mathematical education, I claim that the cultural specificity of these notions should be known and should be fully accounted for in education.

The educational advices I found most appealing in this regard came from a self-declared enemy of Piaget, namely Freudenthal. His main rule denies any claim of apriorism and thus can be used in other cultural contexts. I claim: learning mathematics should basically be done by «having the learner reflect on his learning processes» or, in other words, building up a «mathematical attitude» (Freudenthal, 1981: 141).

In practice, it is thought to be desirable to work with the richest possible examples and projects in the classroom to begin with (e.g. villages, islands, etc... which are constructed at scale model) and to come to the understanding of gradually more general and more abstract notions by focusing first on the concepts that spring to mind in the given context for the child (e.g. Freudenthal, 1979; Five years of IOWO, 1976; Freudenthal, personal communication).

What is proposed in the rest of his paper can be summarized as an extension of the same lines of thought: the cultural as well as the natural context of the children will thus be framed and interpreted in terms of the Navajo tradition. One of the aspects of the tradition (and not a minor one, because of the all-pervasiveness of space in Navajo language, cf. Werner, 1983), is the system of spatial knowledge or spatial differentiations, which is found in the Navajo culture (cf. Part I). I think it is advisable to train the Navajo children in a Navajo context of education and by means of (properly agreed upon) Navajo terms in Navajo visualizations. This process will end in the construction, training and elaboration of abstractions that have a meaning primarily in the Navajo way of thinking (cf. the scope of the research on modern mathematics in Cole et al., 1971).

This focus differs from Piaget's in that the psychological differentiation in children of another culture is granted, and is even brought to a certain degree of sophistication (geometry) before introducing western formal concepts. As far as the choice of an educational paradigm is concerned, I subscribe to one of Piaget's psychological emphases: I am convinced that this stress on action as the basic tool for thought allows for a sufficiently general educational entry. Apostel (1984) already worked this out, for geometry teaching and (1984b) for arithmetic, in an successful attempt to reconcile Piaget's theory and Freudenthal's educational principles.

3. NAVAJOS, TEACHING OF GEOMETRY, RODEO AND OTHER ISSUES

I will give only one example of a curriculum item I developed along this line of thought. The curriculum was used (in 1982-83) as a means to explore geometry and, at the same time, to do research on changes in spatial knowledge. The child explores itself in the classroom setting, or in the open air. I work by means of five projects for geometry teaching, of which the first one will be elaborated inmediately:

1. rodeo; 2. hooghan; 3. school compound; 4. herding sheep; 5. rug weaving.

Each of these projects deals with activities and phenomena the children know personnally. Each one is rich in spatial meaning and can be explored, reconstructed, visualized, described and acted out by children (7-10 years). Therefore, these cultural contexts seem to offer the ideal means to have the children explore the inherent geometry in their understanding of their cultural and physical environment, and thus to gradually develop geometric concepts.

In order to explore these contexts in a way that is close to the child's own experience, I allow for manipulation, active explorations, remaking and the like as much as possible (on this point Freudenthal, 1973, and Piaget, 1969 agree totally, for once). Waste material, geoboard, drawing material, etc., are used.

Project one: the rodeo

Rodeo is a family event. Gradually, rodeos were organized on the reservation, which prescribed that only Indians can compete for the prices (cf. the All Indian Cowboy Rodeo, Ganado, 1977). Navajos are growing to be specialists in cattle roping, wild bull riding, and so on, and the sport is catching on as a major from of amusement.

The children are asked to make a rodeo ground in the classroom or on the school compound:

— An oval central arena is constructed.

— The cubic cages for animals and the elevated speaker's lodge are situated at the edge of the arena.

— Neighboring standing places for the audience are delineated.

— A particular «funnel» system for the entrance of the animals is constructed.

The exploration in the classroom comprised several stages (reported by H. Vandenbogaerde):

— The class paid a visit to two rodeo grounds in the neighborhood: the grounds were motorically explored (touching, climbing, running, etc...). Meanwhile children were asked to draw. The task was: make drawings so you will be able to reconstruct the rodeo ground in the classroom.

— In the classroom they started to reconstruct a maquette of the rodeo three times: 1) a large sized rodeo, so they could all sit in it (carton boxes and sticks, etc.); 2) a medium sized one, allowing for one person to sit in it and having a complete overview of the ground (only carton boxes are used; they are cut to strips and glued together); 3) a small rodeo, so the construction would cover one table (using little frisco-sticks to make fences, cages, a lodge, etc...). The children each time decided to leave a project (1, and then 2) and to attempt to make a smaller scale model. Meanwhile the Navajo labels for all parts were used. Practically no directives were given by the teacher.

From this educational experiment a few observations could be drawn:

— Children had trouble to estimate proportions and decided themselves to diminish the size in order to master proportions.

— There was very poor cooperative action (a typical Navajo social feature), such that there never was a global form (e.g. the «topological» form of an oval, closed arena) to start with. Instead, everybody made a piece of fence or a wall of a building, which was at some point glued to another part. Still based on the previous drawings, everybody seemed to head for the construction of an enclosure (the fenced arena), deliberately choosing to construct the borders (fences) first, and allowing for a non-metric quasi oval open space to be defined by the fences. Thus, the concepts of center-periphery, enclosure and border seemed to be constitutive for the children in their actions of reconstructing the rodeo ground (cf. Apostel's program for an action theoretic instruction of geometry, 1984).

— It was only with the frisco-sticks that they managed to construct a speaker's lodge. The fences were very neat and regular in this version. It is clear that the form and other particularities of the material (equal size, etc.) determined the result to some extent.

— The first version was abandoned also because of technical problems: they were unable to make the knots necessary to rope sticks and carton boxes together in order to construct a fence.

The few results already give a clear indication on what can be meant by the learner's «reflecting on his learning process» (see Freudenthal, supra). I think they point out some specific preferences in the learning strategy of the children of this culture: border and enclosure, but most certainly metric forms also (cube, rectilinear form of the icesticks) are focused. On the other hand, the «topological» notions embedded in knots cannot be mastered.

The general focus on action-directedness (active exploration, the «phrasing» of forms and regularities in actions) in the use of the geometry curriculurm confirms Piaget's insight about the fundamental status of action (cf. again Apostel, 1984; 1984b). However, the necessity to work with the children's culture-specific concepts and strategies of exploration cames out, even in this report of a single part of curriculum development in a cultural perspective. As far as I can see now this point is confirmed in all similar studies we carried through in this program (reported in Pinxten et al., 1987). If this finding can thus be generalized (but this will take a couple of years from now to evaluate fully), it implies that Piaget's model of spatial development can only be rigidly used as a basis for the geometric education of western children at the most. Children from other cultures seem to start out with a different system of pre-school knowledge; they recognize different meaningful contexts for geometry teaching; they develop different strategies to acquire formal thought (albeit they may all share the investment in action) and they develop more or less diverging concepts along the road of development.

Thus, at least one critical remark can be made already: Piaget's interpretation of the content and the logical order of stages of development (Piaget, 1969) might need some modification. That is, the conclusions of «lacking behind» or «not reaching the formal operational level of children in other cultures». (Berry & Dasen, 1974; Darke, 1982) may be erroneous descriptions based on a faulty methodology, whereas the analysis and teaching in an autochtonous perspective yields only «culture» particularistic difference (see also the methodological critique in Cole et al., 1971).

CONCLUDING REMARKS

The rodeo project is one of the five projects used in Navajo geometry teaching. Apart from these, we started similar projects with Turkish immigrant children: the projects will probably take some years to have their full results evaluated properly. However, that much is clear: the present research can be situated in the general revival of intuitive approaches to learning to think. It aims at offering some insights in the cross-cultural relevance of this intuitive approach and at going counter to the more rationalistic and indeed implicity imperialistic tendency to «engineer» the minds of the others towards our particular and highly biased way of seeing and manipulating the world.

We have taken the position in this paper that development is necessary and unavoidable for any human being, but we clearly expressed the view that development can start from an understanding of the autochtonous world view and can be directed towards the «unfolding» of the potentialities of this native perspective, rather than towards an implementation of foreign and alienating avenues of thought. Geometry might be, because of its basic status for human exploration of the world of experience, a crucial area in this respect and its development along autochtonous lines may well amount to be «genuine development».

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